Non-linear Filtration Of Impulse Noise By Means Of Cellular Neural Networks

Elena Solovyeva
Theoretical Electrical Engineering Department
Saint-Petersburg Electrotechnical University “LETI”
Saint-Petersburg, Russia
selenab@hotbox.ru

Abstract—The synthesis of nonlinear impulse noise filters in the form of feedforward cellular neural network and two-layer perceptron network are considered. Neural filters are used for cancelling the “salt and pepper”-type interference from half-tone images. It is shown that the filter in the form of cellular neural network surpasses two-layer perceptron filter in the filtration accuracy. Neural filters ensure a higher level of impulse noise suppression as compared with median filters.

Keywords—nonlinear model; neural network; nonlinear filter; impulse noise; image

I. INTRODUCTION

One of major problems in the communication theory is non-linear filtering non-Gaussian noise. The impulse noise relates to non-Gaussian noise. This noise emerges during the switching of different electronic devices, in cases of mechanical damages of surfaces of the data storage devices, during the operation of internal-combustion engines, under the impact of various atmospheric phenomena, etc. Non-Gaussian noise distorts the working signals. Its emerging leads to the impairment of data transmission in communication channels [1]–[4].

Non-Gaussian noise is not cancelled by the methods of linear signal processing, so the methods of non-linear signal processing are used for non-Gaussian noise suppression, in particular, the methods within the framework of the “black box” principle. In view of this principle, the methods of building multidimensional polynomials and neural networks for non-linear devices, such as transformers, filters, compensators etc., are developed [5]–[7].

In recent years numerous non-linear filters are synthesized in the form of neural networks, which are universal approximators, and in some cases, they are simpler in comparison with multidimensional polynomials [6], [7].

II. THE PROBLEM OF IMPULSE NOISE FILTRATION FOR IMAGE RESTORATION

The problem of synthesizing the digital impulse noise filters can be solved within the framework of the “black box” principle [2], [5], [6], when filter operator \( F \) establishes a unique relationship between the set of input \( s(n) \) \((s(n) \in S)\) and output \( y^o(n) \) \((y^o(n) \in Y^o)\) signals of device

\[
y^o(n) = F[s(n)].
\]

For nonlinear dynamic system modeling it is required to approximate nonlinear operator \( F \) by nonlinear operator \( F_\varepsilon \), which reflects the input set \( X \) on the output set \( Y^o \) with error \( \varepsilon \), \( \varepsilon > 0 \), that is

\[
y(n) = F_\varepsilon[s(n)]
\]

where \( n \in [0,G_n] \) is the normalized discrete time, \( G_n \) is the duration of input signals.

The digital filter synthesis consists in building a mathematical model in the form of operator equation

\[
y(n) = F_\varepsilon[s(n)],
\]

where \( y(n) \) is the output signal of the filter model, \( F_\varepsilon \) is operator approximating \( F \) on sets \( S \) and \( Y^o \) under the condition

\[
\|y^o(n) - y(n)\| \leq \varepsilon
\]

for all \( s(n) \in S \), \( y^o(n) \in Y^o \). Here \( \varepsilon \) is the assigned error of simulation.

The parameters of nonlinear operator \( F_\varepsilon \) are determined by solving the approximation problem

\[
\|y^o(n) - F_\varepsilon[s(n)]\| \rightarrow \min_D,
\]

where \( D \) is the parameter set of operator \( F_\varepsilon \). In practice, the approximation error is usually estimated in the mean-square norm.

The synthesis of combined filters (with an internal smoothing median filter (MF)) for image restoration [8]–[10],...
and application of neural networks as approximators [10] stipulate combined neural filter structure shown in Fig. 1. Here the block MF designates smoothing median filter, the block NN denotes neural network.

\[
\begin{array}{ccc}
\mathbf{s}(n) & \text{MF} & \mathbf{u}(n) \\text{NN} & \mathbf{x}(n)
\end{array}
\]

Fig. 1. The structure of combined neural filter.

The set of signals \(\mathbf{u}(n)\) is interpreted as a set of fragments having length \(m\), each of which is the vector composed of samples of the input signal at the template area (aperture) moving along the image with the step of one cycle.

The neural network (the block NN depicted in Fig. 1) can be specified in the form of cellular neural network.

### III. CELLULAR NEURAL NETWORK

The cellular neural network (CNN) was introduced by Chua and Yang (1988) [11], [12]. It is a recurrent nonlinear network in which neurons are locally connected, and dynamics is identical for each node. These neurons are commonly called cells. The connection with the cells outside the \(r\)-neighborhood is enabled by the propagation effects of network dynamics. The CNN dynamics is described by a set of differential equations [13]–[15].

Each cell of CNN has an input, an internal state and an output. Any one cell is connected only to its neighboring cells. Cell located in the position \((i, j)\) of two-dimensional \(M \times N\) area is denoted as \(C_{ij}\), and its \(r\)-neighborhood \(\mathcal{N}_{ij}^r\) is defined by

\[
\mathcal{N}_{ij}^r = \{ C_{kl} , \max(|k-i|,|l-j|) \leq r, 1 \leq k \leq M; 1 \leq l \leq N \}
\]

where the size of the neighborhood \(r\) is a positive integer number.

Set \(\mathcal{N}_{ij}^r\) is sometimes referred to as the \((2r+1)\times(2r+1)\) neighborhood. For the \(3\times3\) neighborhood, \(r = 1\). Thus, the parameter \(r\) controls the connectivity of a cell, i.e. the number of active synapses that connects the cell with its immediate neighbors.

CNN is entirely characterized by the set of nonlinear differential equations associated with cells in network. The mathematical model for the state equation of a single cell \(C_{ij}\) is given by the following relation:

\[
\frac{\partial x_{ij}(t)}{\partial t} = -x_{ij}(t) + \sum_{k \in \mathcal{N}_{ij}^r} A_{ij,k} y_{kj}(t) + \sum_{k \in \mathcal{N}_{ij}^r} B_{ij,k} u_{kl}(t) + I_{ij},
\]

where \(x_{ij}(t)\) denotes the state of cell \(C_{ij}\); \(y_{kl}(t)\), \(u_{kl}(t)\) denote the output and input of cells \(C_{kl}\) located in the sphere of influence with radius \(r\), respectively; \(A_{ij,k}\) and \(B_{ij,k}\) are the feedback and feedforward templates, respectively; \(I_{ij}\) is the bias term.

In many applications, CNN is isotropic, that is space-invariant. Isotropic network is characterized by parameters in equation (1) which are fixed for the entire neural network. In the case of isotropic CNN, for example under \(r = 1\), the terms of state equation (1) are represented below.

- Contribution of the feedback synaptic weights \(A_{ij,k}\) to equation (2). In view of space-invariance, we can write

\[
\sum_{k \in \mathcal{N}_{ij}^r} A_{ij,k} y_{kl}(t) = \sum_{|k-i| \leq 1} \sum_{|j-l| \leq 1} A(i-k, j-l) y_{kl}(t) =
\]

\[
= \sum_{k=-1}^{1} \sum_{l=-1}^{1} a_{k,l} y_{i+k,l+j} =
\]

\[
= \begin{bmatrix}
    a_{-1,-1} & a_{-1,0} & a_{-1,1} \\
    a_{0,-1} & a_{0,0} & a_{0,1} \\
    a_{1,-1} & a_{1,0} & a_{1,1}
\end{bmatrix} \odot
\begin{bmatrix}
    y_{i-1,j-1} & y_{i-1,j} & y_{i-1,j+1} \\
    y_{i,j-1} & y_{i,j} & y_{i,j+1} \\
    y_{i+1,j-1} & y_{i+1,j} & y_{i+1,j+1}
\end{bmatrix} =
\]

\[
= \mathbf{A} \otimes \mathbf{Y}_{ij},
\]

where the matrix \(\mathbf{A}\) of the \(3\times3\) dimension is called the feedback cloning template, the symbol \(\otimes\) denotes the summation of dot products (or the sign of vector product), henceforth called a template dot product. In discrete mathematics, this operation is called “spatial convolution.”

The \(3\times3\) matrix \(\mathbf{Y}_{ij}\) in (2) can be obtained by moving an opaque mask with the \(3\times3\) window to the position \((i, j)\) of the \(M \times N\) output image \(Y\), henceforth called the output image at \(C(i, j)\).

An element \(a_{kl}\) is called the center (respectively, surround) element, the weight or coefficient, of the feedback template \(\mathbf{A}\), if and only if \((k, l) = (0, 0)\) (respectively, \((k, l) \neq (0, 0)\)).

- Contribution of the input synaptic weights \(B_{ij,k}\) to equation (2). Following the above notes, we can write

\[
\sum_{k \in \mathcal{N}_{ij}^r} B_{ij,k} u_{kl}(t) = \sum_{|k-i| \leq 1} \sum_{|j-l| \leq 1} B(i-k, j-l) u_{kl}(t) =
\]

\[
= \sum_{k=-1}^{1} \sum_{l=-1}^{1} b_{k,l} u_{i+k,l+j} =
\]
\[
\begin{bmatrix}
\tilde{b}_{i-1, 1} & \tilde{b}_{i, 0} & \tilde{b}_{i+1, 1} \\
\tilde{b}_{i-1, 0} & \tilde{b}_{i, 0} & \tilde{b}_{i+1, 0} \\
\tilde{b}_{i-1, -1} & \tilde{b}_{i, -1} & \tilde{b}_{i+1, -1}
\end{bmatrix}
\begin{bmatrix}
u_{i-1, j-1} & u_{i-1, j} & u_{i+1, j+1}
\end{bmatrix} = B \odot U_{ij},
\]

where the \(3 \times 3\) matrix \(B\) is called the feedforward or input cloning template, and \(U_{ij}\) is the translated masked input image.

- Contribution of the threshold term to equation (2). In view of space-invariance, denote \(\tilde{I}_{ij} = z\).

Using the above notations in (3), (4), space-invariant CNN is completely described by state equation

\[
\dot{x}_{ij} = -x_{ij} + A \otimes Y_{ij} + B \otimes U_{ij} + z,
\]

The output signal of cell \(C_{ij}\) is given by the following equation

\[
y_{ij}(t) = f(x_{ij}(t))
\]

where \(y_{ij}(t)\) denotes the output value of cell \(C_{ij}\), \(f(\bullet)\) is the non-linear activation function which is usually specified as the unity gain piecewise linear saturation function described by expression

\[
y_{ij}(t) = f(x_{ij}(t)) = \frac{1}{2} \left( |x_{ij}(t)| + 1 - |x_{ij}(t) - 1| \right)
\]

and shown in Fig. 2.

![Fig. 2. The piecewise linear saturation function.](image)

A significant CNN feature is that CNN has two independent input capabilities: the generic input and the initial state of cells. They are normally bounded by \(|\mu_{ij}(t)| \leq 1\) and \(|x_{ij}(0)| \leq 1\). Similarly, if \(|f(t)| \leq 1\) then \(|x_{ij}(t)| \leq 1\).

CNN is uniquely defined by three terms of the cloning templates \(\{A, B, z\}\), which consist of 19 real numbers for the \(3 \times 3\) neighborhood \((r = 1)\).

One of the simplest CNN subclasses is zero-feedback (feedforward) CNN. CNN belongs to the zero-feedback subclass if and only if all the feedback template elements are zero, i.e., \(A \equiv 0\). In view of (5) each cell of the zero-feedback CNN is described by expression

\[
x_{ij} = -x_{ij} + B \odot U_{ij} + z
\]

The discrete CNN model is used for image processing. The feedforward CNN description in discrete time domain results from expression (7) after following transformations:

- the approximation of derivative

\[
\frac{dx_{ij}(t)}{dt} = \frac{x_{ij}(t) - x_{ij}(t - \Delta t)}{\Delta t} = x_{ij}(n) - x_{ij}(n - 1)
\]

where \(n\) is the discrete normalized time. Let us suppose, that \(\Delta t = 1\);

- the transition from differential equation (6) to recursive difference equation

\[
x_{ij}(n) - x_{ij}(n - 1) = -x_{ij}(n - 1) + B \otimes U_{ij} + z
\]

Eventually, on the bases of (6) and (7) the model of cell \(C_{ij}\) in feedforward discrete-time CNN (DTCNN) is described as

\[
x_{ij}(n) = B \otimes U_{ij} + z,
\]

\[
y_{ij}(n) = f(x_{ij}(n))
\]

The structure of cell \(C_{ij}\) in feedforward DTCNN is depicted in Fig. 3.

![Fig. 3. The structure of cell \(C_{ij}\) in feedforward DTCNN.](image)

Expressions (8) are transformed into the isotropic model of DTCNN cell if the parameters of model are fixed for entire neural network.

IV. THE SIGNALS OF IMAGES AND THE CRITERION OF FILTRATION ACCURACY ESTIMATION

Combined neural filters are synthesized on the class of bit-map (dot element) half-tone images at the resolution measured by 256 gray levels, i.e., image is the matrix of integers
(elements of brightness, pixels) in the interval \([0; 255]\). In the case under consideration, the pixel format is unit8.

The impulse noise represents switched on and switched off pixels (white and black dots in the picture), the emergence of which does not depend on the presence of noise spikes in adjacent dots. The addition of impulse interference to image implies that value \(q\) of the signal sample with probability \(P_a\) is replaced with value \(z = 0\) (black), with probability \(P_b\) is replaced with value \(z = 255\) (white), and with probability \((1-(P_a+P_b))\) remains unchanged. Thus, the probability density of the impulse noise is described by the following expression

\[
p(q) = \begin{cases} 
P_a & \text{at } q = 0; \\
P_b & \text{at } q = 255; \\
0 & \text{in other cases}
\end{cases}
\]

or

\[
p(q) = P_a \delta(q-a) + P_b \delta(q-b),
\]

where \(\delta()\) is the \(\delta\)-function. Let us assume that \(P_a = P_b\). The impulse noise model described above is referred to as “salt and pepper” [4], [9], [10].

In building operator \(F_\varepsilon\) of nonlinear filter, the “unit8” format of signals \(u(n) \in U\) and \(y^o(n) \in Y^o\) is transformed into the “double” format (samples of signals are normalized floating-point numbers of double precision in the range \([-1; 1]\)).

The synthesis of combined neural filter (Fig. 1) with internal isotropic feedforward DTCNN, including some neurons in hidden layer and referred to as combined DTCNN (CDTCNN), is performed in the MATLAB system environment. This synthesis involves solving the approximation problem (1) in the mean-square norm using the error back propagation algorithm [6]. Activation function \(f\) is specified in the form of piecewise linear saturation one (Fig. 2). The distorted image “Tigers” having the size of 220×148 pixels is a learning signal.

The results of CDTCNN filtration are compared with the results of combined neural filtration (Fig. 1) with internal two-layer perceptron network (TLPN), including the hyperbolic tangent activation functions, referred to as combined TLPN (CTLPN) [10], as well as of median filtration performed at the 3x3 square aperture [4].

CDTCNN and CTLPN comprise different activation functions. The properties of the piecewise linear saturation function, shown in Fig. 2 and included in CDTCNN, are the following:

- an uniform gain for low and large input signal amplitudes,
- quickness of signal conversion,
- implementation simplicity in the analog and digital fields using operational amplifiers and VLSI technology respectively.

The properties of hyperbolic tangent activation function, which is shown in Fig. 4 and contained in CTLPN, are the following:

- gain control for the input signals of different levels. The central function part corresponding to the region of low input signal amplitudes has a large slope and a maximum of the derivative, so the gain is maximum here. Moving from the function central to large absolute values of inputs, the slope of the curve and its derivative is decreased, as well as the gain is reduced;
- the function is continuous and differentiable over the entire range of argument, that is convenient on using this function in the gradient algorithms of learning network when multiple operations of differentiation are required;
- the slow conversion of signal;
- more complicated implementation as compared with the piecewise linear function.

![Fig. 4. The hyperbolic tangent activation function.](image)

The rate of impulse noise suppression by different filters is estimated with the help of error \(\varepsilon\) computed after the restoration of a test image having the size of 220×148 pixels by the following formula

\[
\varepsilon = \frac{1}{Q} \sum_{n=1}^{Q} \left( y(n) - y^o(n) \right)^2,
\]

where \(y(n)\) is the output signal of non-linear filter, \(y^o(n)\) is desirable signal, \(Q = 32 560\). The test images differ from learning one.

V. IMPULSE NOISE CANCELLING BY NEURAL FILTERS

The filtration results (error calculated from (9)) obtained by synthesized devices are summarized in Table I and Table II under various densities \(p\) of the impulse noise.

“Tigers”, “Building” and “Fence” are names of learning and two tests images correspondently. All of images have the size of 220×148 pixels.

Non-linear filters built on the base of CDTCNN and CTLPN comprise two and five neurons in hidden layer under
the impulse noise density \( p = 0.3 \) and \( p = 0.5 \) correspondingly. The filtration errors are decreased very slowly at neuron numbers higher than mentioned ones.

<table>
<thead>
<tr>
<th>Image</th>
<th>CDTCNN</th>
<th>CTLPN</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tigers</td>
<td>514</td>
<td>516</td>
<td>739</td>
</tr>
<tr>
<td>Building</td>
<td>786</td>
<td>804</td>
<td>1039</td>
</tr>
<tr>
<td>Fence</td>
<td>1142</td>
<td>1138</td>
<td>1434</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image</th>
<th>CDTCNN</th>
<th>CTLPN</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tigers</td>
<td>771</td>
<td>814</td>
<td>2759</td>
</tr>
<tr>
<td>Building</td>
<td>1083</td>
<td>1186</td>
<td>3014</td>
</tr>
<tr>
<td>Fence</td>
<td>1558</td>
<td>1800</td>
<td>3735</td>
</tr>
</tbody>
</table>

The following inferences can be made from Table I and Table II.

- At the middle noise density (\( p = 0.3 \)), CDTCNN and CTLPN filters comprise two neurons in hidden layer, these filters ensure virtually identical accuracy of filtration. Thus, differences in the activation functions of these filters are not revealed at low number of neurons.
- At the high noise density (\( p = 0.5 \)), when CDTCNN and CTLPN contain five neurons in hidden layer, CDTCNN with the piecewise linear activation functions yields higher filtration precision, than CTLPN with the hyperbolic tangent activation functions.

Indeed, at an equal probability of the impulse noise (white and black dots) occurrence, the filtration with different gains at small and high amplitudes of signals (in the case of the hyperbolic tangent and the logistic activation function) is not expedient. The use of the hyperbolic tangent activation functions in CTLPN negatively affects the image quality (white color turns to gray one, as well as there is a bit ripple i.e. image loses its smoothness).

**CONCLUSION**

The problem of non-Gaussian noise filter synthesis is often effectively solved within the framework of the "black box" principle. According to this principle, the mathematical filter model describes the relationship between the sets of input and output signals. The model parameters are determined by solving the approximation problem using the subsets of input and output signals.

Considered approach to the synthesis of nonlinear filters is general because it can be applied at various kinds of non-Gaussian noise sources.

Neural filters are synthesized for the suppression of the impulse noise such as “salt and pepper” on half-tone images. It is shown that the filters in the form of the feedforward cellular neural network in the standard version (with the piecewise linear saturation functions) carries out more accurate restoration of images, as compared with the two-layer perceptron network (with the hyperbolic tangent or the logistic activation function).

**ACKNOWLEDGMENT**

This work was supported by Saint Petersburg Electrotechnical University “LETI” according to the base part of the state scientific work from the Russian Education Ministry.

**REFERENCES**